Chapter 2: Probability
Learning Objectives

Some courses may choose to skip probability and continue onto Chapter 3 or jump to Sections 7.1-7.3.

Reading: Section 2.1 of OpenIntro Statistics

Video: Basics of probability, YouTube (1:42)

Video: Union of events and the addition rule, YouTube (3:37)

Video: Independent events, intersection of events, multiplication rule, and Bayes' Theorem, YouTube (3:25)

1. Define trial, outcome, and sample space.

2. Explain why the long-run relative frequency of repeated independent events settle down to the true probability as the number of trials increases, i.e. why the law of large numbers holds.

3. Distinguish disjoint (also called mutually exclusive) and independent events.
   - If A and B are independent, then having information on A does not tell us anything about B.
   - If A and B are disjoint, then knowing that A occurs tells us that B cannot occur.
   - Disjoint (mutually exclusive) events are always dependent since if one event occurs we know the other one cannot.

4. Draw Venn diagrams representing events and their probabilities.

5. Describe properties of probability distributions.

6. Define complementary events as two events whose probabilities add up to 1.

7. Distinguish between union of events (A or B) and intersection of events (A and B).
   - Calculate the probability of the union of events using the general addition rule.
   - Calculate the probability of intersection of independent events using the multiplication rule.

Test yourself:

1. What is the probability of getting a head on the 6th coin flip if in the first 5 flips the coin landed on a head each time?
2. True / False: Being right handed and having blue eyes are mutually exclusive events.
3. P(A) = 0.5, P(B) = 0.6, there are no other possible outcomes in the sample space. What is P(A and B)?

Reading: Section 2.2 of OpenIntro Statistics

Video: Probability trees, Dr.Çetinkaya-Rundel (8:23)

Video: Conditional probability, YouTube (8:59 - watch from 3:33 onwards)

Video: Bayes' Theorem worked out example, YouTube, (9:20, somewhat lengthy)

Video: Another example of conditional probabilities using Bayes' Theorem, YouTube (7:20)

8. Distinguish between marginal and conditional probabilities.

9. Use tree diagrams and/or Bayes Theorem to calculate conditional probabilities and probabilities of intersection of non-independent events.

Test yourself: 50% of students in a class are social science majors and the rest are not. 70% of the social science students and 40% of the non-social science students are in a relationship. Create a contingency table and a tree diagram summarizing these probabilities. Calculate the percentage of students in this class who are in a relationship.

Reading: Section 2.3 of OpenIntro Statistics

10. When sampling from a small population without replacement, the draws are not independent.

Test yourself: What is the probability of drawing, without replacement, a queen, a jack, and a king from a full well-shuffled deck, in that order? What is the probability of drawing the same hand with replacement?

Reading: Section 2.4 of OpenIntro Statistics

11. A random variable is a variable or process with a numerical outcome.

12. The expected value of a discrete random variable is computed by adding each outcome weighted by its probability.

$$E(X) = \mu = \sum_{i=1}^{k} x_i P(X = x_i)$$

13. The variance of a discrete random variable is computed by adding each squared deviation of an outcome from the expected value weighted by its probability. The standard deviation is the square root of the variance.

$$Var(X) = \sigma^2 = \sum_{i=1}^{k} (x_i - \mu) P(X = x_i)$$

14. The average of a linear combination of discrete random variables is computed as the sum of their averages, weighted by the constant multipliers.

$$E(aX + bY) = a \times E(X) + b \times E(Y)$$

15. The variance of a linear combination of independent discrete random variables is computed as the sum of their variances, weighted by the square of the constant multipliers.

$$Var(aX + bY) = a^2 \times Var(X) + b^2 \times Var(Y)$$

Reading: Section 2.5 of OpenIntro Statistics

16. The distribution of a continuous random variable is described by the probability density function.

17. The total area under the density curve is 1.

18. Probabilities under the density curve can be calculated as the area under the curve.

19. The probability of a continuous random variable being exactly equal to a value is 0, since there is no area under the curve at a given location.